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State Dependent M/M/1 Queue with Unreliable Server, Second Optional Service in Fixed Batch, Vacation and Setup

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Abstract

Present paper deals with state dependent, unreliable M/M/1 queueing system with server vacation, setup and second optional service in batches. The single server apart from providing the usual service one by one also provides an additional optional service to the customers in batches of fixed size $b(b \geq 1)$. The customers are queued up for first service which is essential for all the customers. The second service is optional which is demanded by some of the customers whereas the others leave the system after the first service. The server takes single vacation each time the system becomes empty. The server may breakdown during the service and goes for repair immediately. We have constructed steady state equations by considering state dependent in-flow and out-flow rates for different states. By applying probability generating function technique we determine the probability of empty system, expected number of units in the system.

Keywords: Optional batch service, State Dependent Arrival, Vacations, Setup time, Waiting time, Single vacation.

1. Introduction

In this paper, state dependent M/M/1 queueing system is considered with server vacation, setup and second optional service in batches. There is provision of single server, apart from providing the usual service one by one, server also provides an additional optional service to the customers in batches of fixed size $b(b \geq 1)$. The customers are queued up for first service which is essential for all the customers. The second service is optional which is demanded by some of the customers whereas the others leave the system after the first service. The server takes single vacation each time the system becomes empty also the server may breakdown during the service and goes for repair immediately.

Operating characteristic of M/M/1 queueing system under N-policy with exponential set up time has been provided by Choudhary (2001). Queueing model with backup servers and service breakdowns was specified by Gray et al. (2002). The optimal control of M/G/1 queueing system with server vacations, startup and breakdowns was suggested by Ke (2003). Optimal management policy for heterogeneous arrival queueing system with server breakdowns and vacations was provided by Ke and Pearn (2004). Diaz and Moreno (2009) studied a queueing system where the service station operates under an N-policy with early setup.

The organization of paper is as follows: the model under consideration is described in section 2. In section 3, we examine the system by using probability generating technique and obtain some operating characteristics in section 4. Section 5 is devoted to numerical analysis. Conclusion is drawn in section 6.

2. Model Description

Consider a single non-reliable removable server Markovian queueing model with server vacation, setup and second optional service in batches. Let the state i= 0 represents the state when server is on vacation, i=1, when server is working; i=2, when server is found to be broken down; i=3, when server is under repair. We assume that customer's arrival follows a Poisson process with rates λ_i (i=0,1,2,3) where 0,1,2,3 denote the arrival rates of customers during the idle, busy, breakdown and repair period, respectively.

The server may serve only one unit at a time and the service rates are exponentially distributed with mean $1/\mu$. Whenever the system is empty the server goes for vacation. The duration of vacations is exponentially distributed with mean $1/\theta$. Duration of each vacation is independent of arrival process, the service time and breakdown times. During the service the server may breakdown at any time with Poisson breakdown rate α. The setup time to initiate the repair is exponentially distributed with rate ν. When the server fails, it is immediately repaired with repair rate β. Once the repair of the server is completed, it immediately starts to provide service.

Following notations and probabilities are used throughout the paper for formulating the model mathematically:

λ_i(i=0,1,2,3) Arrival rate of customers in various status

μ Mean service rate of the server

θ Mean vacation period

$P_0(n)$ Probability that server is on vacation.

$P_1(n)$ Probability that server is working

$P_2(n)$ Probability that server is found to be broken down.

$P_3(n)$ Probability that server is under repair.

$H_i(z)$ The probability generating function of $P_i(n)$, i=0,1,2,3

$E(N_i)$ Expected number of costumers in the system when the server is in the state i, i=0,1,2,3.

Governing Equations:

Steady state equations governing the model are given as follows:

$$(\lambda_1+\mu_1)P^{(1)}_{m,n}=\lambda_1 P^{(1)}_{m-1,n}+\mu_1 q P^{(1)}_{m+1,n}+\mu_1 p P^{(1)}_{m+1,n-1};\ m>0, 0<n<b \quad (1)$$

$$(\lambda_1+\mu_1)P^{(1)}_{m,0}=\lambda_1 P^{(1)}_{m-1,0}+\mu_1 q P^{(1)}_{m+1,0}+\mu_2 P^{(2)}_{m+1,0}+\theta Q_{m+1,0};\ m>0 \quad (2)$$

$$(\lambda_1+\mu_1)P^{(1)}_{0,n}=\mu_1 q P^{(1)}_{1,n}+\mu_1 p P^{(1)}_{1,n-1};\ 0<n<b \quad (3)$$

$$(\lambda_1+\mu_1)P^{(1)}_{0,0}=\mu_1 q P^{(1)}_{1,0}+\mu_2 P^{(2)}_{1,0}+\theta Q_{1,0}+(\lambda_0+\vartheta) \quad (4)$$

$$(\lambda_2+\mu_2)P^{(2)}_{m,0}=\lambda_2 P^{(2)}_{m-1,n}+\mu_1 p P^{(1)}_{m,b-1};\ n=b, m>0 \quad (5)$$

$$(\lambda_2+\mu_2)P^{(2)}_{0,0}=\mu_1 p\sum_{n=1}^{b-1}P^{(1)}_{0,n};\ n=b \quad (6)$$

$$(\lambda_3+\theta)Q_{m,0}=\lambda_3 Q_{m-1,0};\ m>0 \quad (7)$$

$$(\lambda_3+\theta)Q_{0,0}=\mu_1 q P^{(1)}_{0,0}+\mu_2 P^{(2)}_{0,0} \quad (8)$$

$$(\lambda_0+\vartheta)P^{(0)}_{0,0}=\theta Q_{0,0} \quad (9)$$

3. Probability Generating Functions

We define the generating functions as follow:

$$P^{(1)}_n(\alpha)=\sum_{m=0}^{\infty}P^{(1)}_{m,n}\alpha^m;$$

$$P^{(1)}_n(\alpha,\beta)=\sum_{m=0}^{\infty}P^{(1)}_m(\beta)\alpha^m=\sum_{n=0}^{b-1}P^{(1)}_n(\alpha)\beta^n=\sum_{n=0}^{b-1}\sum_{m=1}^{\infty}P^{(1)}_{m,n}\alpha^m\beta^n;$$

$$P^{(1)}_m(\beta)=\sum_{n=1}^{b-1}P^{(1)}_{m,n}\beta^n; \qquad P^{(2)}_0(\alpha)=\sum_{m=0}^{\infty}P^{(2)}_{m,0}\alpha^m;$$

$$Q_0(\alpha)=\sum_{m=0}^{\infty}Q_{m,0}\alpha^m$$

Operating equation (1) by $\sum_{m=1}^{\infty}\alpha^{m+1}$ and equation (3) by α, and then adding for all possible values of m, we get:

$$\{(\lambda_1(1-\alpha)+\mu_1)\alpha-\mu_1 q\}P^{(1)}_n(\alpha)=\mu_1 p P^{(1)}_{n-1}(\alpha)-\mu_1 q P^{(1)}_{0,n}-\mu_1 p P^{(1)}_{0,n-1} \quad (10)$$

Operating equation (2) by $\sum_{m=1}^{\infty}\alpha^{m+1}$ and equation (4) by α, and then adding for all possible values of m, we get:

$$\{(\lambda_1(1-\alpha)+\mu_1)\alpha-\mu_1 q\}P^{(1)}_0(\alpha)=\mu_2 P^{(2)}_0(\alpha)+\theta Q_0(\alpha)-(\lambda_3+\theta)Q_{0,0}-(\alpha-1)(\lambda_0+\vartheta)P^{(0)}_{0,0} \quad (11)$$

Operating equation (5) by $\sum_{m=1}^{\infty}\alpha^m$ + equation (6), we get:

$$\{(\lambda_2(1-\alpha)+\mu_2)\}P^{(2)}_0(\alpha)=\mu_1 p P^{(1)}_{b-1}(\alpha)+\mu_1 p\sum_{n=1}^{b-2}P^{(1)}_{0,n} \quad (12)$$

Operating equation (7) by $\sum_{m=1}^{\infty}\alpha^m$ + equation (8), we get:

$$\{(\lambda_3(1-\alpha)+\theta)\}Q_0(\alpha)=(\lambda_3+\theta)Q_{0,0} \quad (13)$$

Operating equation (10) by $\sum_{n=1}^{b-1}\beta^n$ + equation (11), on simplifying we get:

$$P^{(1)}(\alpha,\beta)=\frac{\mu_2 P_0^{(2)}(\alpha)+\theta Q_0(\alpha)-(\lambda_3+\theta)Q_{0,0}-(\alpha-1)(\lambda_0+\vartheta)P_{0,0}^{(0)}-\mu_1(q+p\beta)P_0^{(1)}(\beta)+\mu_1 q P_{0,0}^{(1)}}{\{(\lambda_1(1-\alpha)+\mu_1)\alpha-\mu_1(q+p\beta)\}}$$

(14)

For $\beta=0$, $P^{(1)}(\alpha,\beta)=P_0^{(1)}(\alpha)$, $P_0^{(1)}(0)=P_{0,0}^{(1)}$, thus for $\beta=0$ equation (14) gives

$$P_0^{(1)}(\alpha)=\frac{\mu_2 P_0^{(2)}(\alpha)+\theta Q_0(\alpha)-(\lambda_3+\theta)Q_{0,0}-(\alpha-1)(\lambda_0+\vartheta)P_{0,0}^{(0)}}{\{(\lambda_1(1-\alpha)+\mu_1)\alpha-\mu_1 q\}} \quad (15)$$

For $\beta=1$ equation (12) gives

$$P_0^{(2)}(\alpha)=\frac{\mu_1 p P_0^{(1)}(\alpha)}{(\lambda_2(1-\alpha)+\mu_2)}$$

(16)

Substitute equations (13) and (16) in equation (15), we get

$$P_0^{(1)}(\alpha)=\frac{(\lambda_2(1-\alpha)+\mu_2)\left\{(\lambda_3+\theta)\lambda_3 Q_{0,0}-(\lambda_0+\vartheta)(\lambda_3(1-\alpha)+\theta)P_{0,0}^{(0)}\right\}}{(\lambda_3(1-\alpha)+\theta)\left[\{(\lambda_1(1-\alpha)+\mu_1)\alpha-\mu_1 q\}(\lambda_2(1-\alpha)+\mu_2)-\mu_1\mu_2 p\right]} \quad (17)$$

By using equation (16) and (17) we get

$$P_0^{(2)}(\alpha)=\frac{\mu_1 p\left\{(\lambda_3+\theta)\lambda_3 Q_{0,0}-(\lambda_0+\vartheta)(\lambda_3(1-\alpha)+\theta)P_{0,0}^{(0)}\right\}}{(\lambda_3(1-\alpha)+\theta)\left[\{(\lambda_1(1-\alpha)+\mu_1)\alpha-\mu_1 q\}(\lambda_2(1-\alpha)+\mu_2)-\mu_1\mu_2 p\right]} \quad (18)$$

Let $P_q(\alpha)$ be the probability generating function of the number of customers in the queue, thus we define

$$P_q(\alpha)=P_{0,0}^{(0)}+Q_0(\alpha)+P_0^{(1)}(\alpha)+P_0^{(2)}(\alpha)$$

(19)

Substitute equations (13), (17) and (18) in (19), on simplification we get,

$$P_q(\alpha)=\frac{N(\alpha)}{D(\alpha)}P_{0,0}^{(0)}$$

(20)

where

$$N(\alpha)=(\lambda_3(1-\alpha)+\theta)\theta+(\lambda_3+\theta)(\vartheta+\lambda_0)+(\lambda_3(1-\alpha)+\theta)(\lambda_2(1-\alpha)+\mu_2)(\vartheta+\lambda_0)\{(\lambda_3+\theta)\lambda_3 -\theta(\lambda_3(1-\alpha)+\theta)\}+\mu_1 p(\lambda_3(1-\alpha)+\theta)(\vartheta+\lambda_0)\{(\lambda_3+\theta)\lambda_3-\theta(\lambda_3(1-\alpha)+\theta)\}$$

$$D(\alpha)=\theta(\lambda_3(1-\alpha)+\theta)^2\{\{(\lambda_1(1-\alpha)+\mu_1)\alpha-\mu_1 q\}\{\lambda_2(1-\alpha)+\mu_2\}-\mu_1\mu_2 p\}$$

To establish $P_{0,0}^{(0)}$, substitute $\alpha=1$ in equation (20), we have

$$P_{0,0}^{(0)}=\left[\frac{\theta^2}{(\lambda_3+\theta)(\vartheta+\lambda_0)+\theta^2}\right]\left[1-\frac{\lambda_0}{\mu_1}\left(\frac{p\mu_1}{\mu_2}+1\right)\right]$$

(21)

steady state condition is $\frac{\lambda_0}{\mu_1}\left(\frac{p\mu_1}{\mu_2}+1\right)<1$

(22)

4. Operating Characteristics

Let $E(L_q)$ denote the mean number of customers in the queue. The average queue length is given by

$$E(L_q)=\frac{d}{d\alpha}[P_q(\alpha)]_{\alpha=1}=\frac{D(1)N'(1)-N(1)D'(1)}{[D(1)]^2}P_{0,0}^{(0)}$$

(23)

Let $E(W_q)$ denote the average waiting time of a customer in the queue, then by using Little's formula $E(L_q)=\lambda E(W_q)$, we get

$$E(W_q)=\frac{D(1)N'(1)-N(1)D'(1)}{\lambda[D(1)]^2}P_{0,0}^{(0)}$$

Where

$$N(1)=\mu_1\mu_2\left[\frac{(\lambda_3+\theta)(\vartheta+\lambda_0)}{\theta}+\theta\right]$$

$$D(1)=\theta\mu_1\mu_2\left[1-\frac{\lambda_0}{\mu_1}\left(\frac{p\mu_1}{\mu_2}+1\right)\right]$$

$$N'(1)=-\mu_1\mu_2(\vartheta+\lambda_0)0-\left[\frac{(\lambda_3+\theta)(\vartheta+\lambda_0)}{\theta}+\theta\right](\lambda_3+\theta)(\mu_1+\mu_2)$$

$$D(1)=(\lambda_3+\theta)\mu_1\mu_2\left[1-\frac{\lambda_0}{\mu_1}\left(\frac{p\mu_1}{\mu_2}+1\right)\right]+\theta(\vartheta+\lambda_0)(\lambda_3-\mu_1-\mu_2)$$

5. Sensitivity Analysis

To explore the effect of different parameters on performance, we perform numerical experiments by taking illustrations.

Figures 1-4 depict the expected number of customers in the system E(N) by varying arrival rate (λ), service rate (μ), breakdown rate (α),respectively for the following sets of arrival rates:

Set 1: $\lambda_0=\lambda_1=\lambda_2=\lambda_3=\lambda$

Set 2: $\lambda_0=\lambda$, $\lambda_1=1.4\lambda$, $\lambda_2=0.9\lambda$, $\lambda_3=0.7\lambda$

Set 3: $\lambda_0=\lambda$, $\lambda_1=1.2\lambda$, $\lambda_2=\lambda$, $\lambda_3=0.8\lambda$

In figure 1 as we increase the threshold level, the queue length increases linearly. Figure 2 shows the gradually increment initially and then after there is a sharp increment in E(N) as arrival rate increases. In figure 3 we exhibit the graphs for E(N) and notice that it increases with the increase in α. Fig 4 displays that as we increase μ, we see that initially average queue length decreases sharply and then becoming almost constant.

Thus we conclude that arrival rates, service rates, breakdown rates and N affect the average queue length differently as per physical expectations for example. With high traffic intensity the queue length increases while by increasing the service rate the queue length decreases.

6. Conclusion

In this paper, we have developed steady state performance indices for N-policy M/M/1 queueing system with server breakdowns, vacations and setup time. We have derived the distribution of system size and employed the probability generating function technique to obtain mean queue length. Many existing queueing models are deduced as special cases of our queueing model. Our queueing model accommodates the real world congestion situations more closely in comparison to other similar studies done previously. Sensitivity analysis performed to examine the effect on the average queue length and cost function of different parameters, may be helpful to decision makers and system designers for the choice of optimal control policy.

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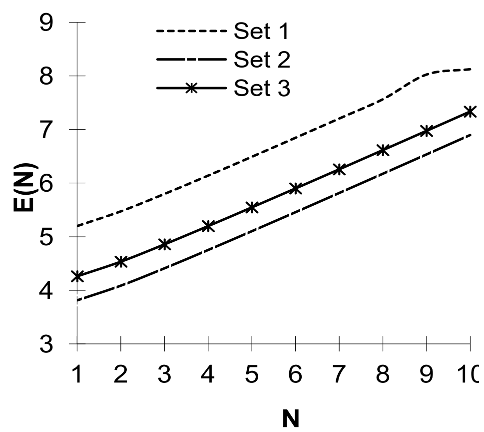

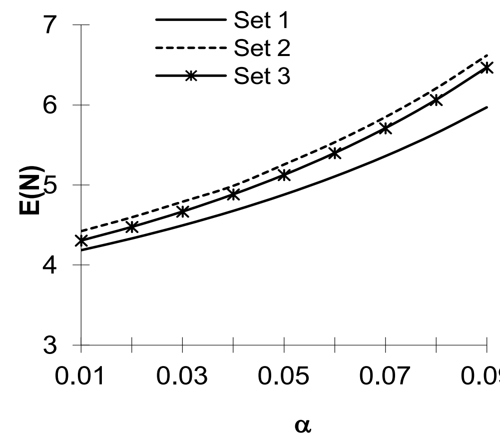

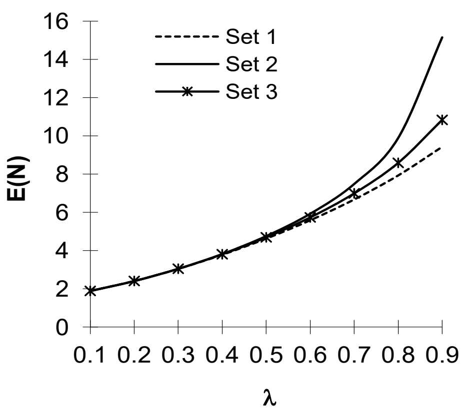

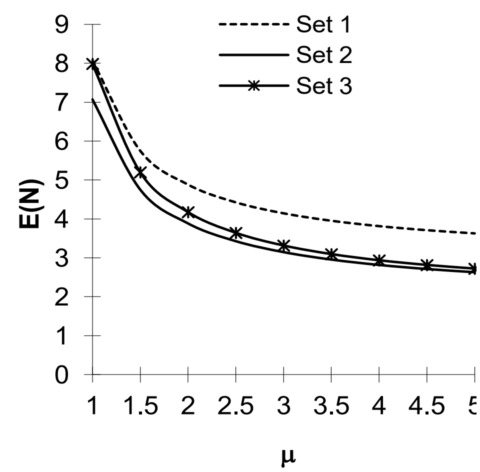

Fig. 1 Expected queue length vs. N

Fig. 3 Expected queue length vs. α

Fig. 2 Expected queue length vs. λ

Fig. 4 Expected queue length vs. μ